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Static Sphere Of Dust Of Uniform Density Using Isotropic Line Element

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ABSTRACT

We present an exact solution of Einstein's field equation in the case of static sphere of dust of uniform density. This is achieved by putting pressure P =0, in Wyman's1 exact solution, for a static sphere of fluid of uniform density. According to Newton's theory, there cannot be a static sphere of fluid of uniform density, hence it is suggested that the term ma (mass of ath particle) in Hilbert's2 action, is not a constant of motion, as has been assumed by Hilbert, but a function of coordinates. It is also suggested that the conditions of Newtonian approximation of Einstein's field equation, given by Eddington3 are the constraints under which Einstein's field equation holds good.

Keywords : Dust, Action (Hamilton's Principle of Least Action), Constraints.

INTRODUCTION:

Wyman has obtained static solutions for a perfect fluid of uniform density using isotropic line element

$$ds = -e^{\mu}\left(dr^2 + r^2 d\theta^2 + r^2 \sin^2\theta - d\varphi^2\right) + e^{v} dt^2$$ ------ (1)He has obtained the solution of (4, 4) equation as

$$e^{\mu} = \frac{4R^2}{\left(e^c r^2 + e^{-c}\right)^2}$$ -------- (2)

With $R^2 = \frac{3}{8\pi\rho}$ and where $\tilde{n}_0$ is proper density, c is an arbitrary constant,

While $\tilde{n} = \tilde{n}_0 + 3p$ is regarded as a constant in his analysis. In the case of a static sphere of dust of uniform density, p=0, so one gets the same solution (2) with with $R^2 = \frac{3}{8\pi\rho}$.

Wyman has eliminated pressure from the other two field equations by subtraction, so as to get an equation in $í'', í', ì'', ì'$. On substituting $e^{ì}$ as given in above equation (2) he has obtained $e^{í}$ as

$$e^{\nu} = \left[\frac{\left(Ar^2 + B\right)}{e^c r^2 + e^{-c}}\right]^2$$ ------- (3)

where A and B are constants of integration.

Wyman has given the exterior solution as

$e^{í} = \frac{\left[q - \left(\frac{m}{r}\right)\right]^2}{q^2}$ and $e^{ì} = q^4$

where $q = \left(1 + \frac{m}{2r}\right)$, 'r' is the radius of the sphere, 'm' is a constant of integration and is interpreted as the total energy of the system. For dust, solution (4) remains unaltered but 'm' now is a constant which means total energy of dust. The constants A and B in equation (3) and constant 'c' in equation (2) are determined by matching v,v′ and μ at the boundary r = a Wyman finally gives $e^{í}$ and $e^{í}$ in terms of m. One gets the same solutions in the case of dust with the only change that m is now the total energy of dust.

Wyman has found pressure p as a function of r and used the boundary condition p = 0 at r = a. In the case of dust p = 0, so that boundary condition is satisfied for dust and as we have stated earlier the constant $\tilde{n}_0 + 3p$ for fluid is replaced by $\tilde{n}_0$ for dust.

Comparison with Newton's theory shows that Einstein's field equation can predict static sphere of dust of uniform density and this is impossible by Newton's theory. In the case of fluid, there is a "pressure" to oppose the attraction due to gravity. In dust there is no pressure to oppose the attraction due to gravity. Hence, a sphere of dust must collapse according to Newton.

NEWTONIAN APPROXIMATION:

Newtonian approximation is a must in the case of weak fields and low velocities. This is because Newton's law of gravitation and his laws of motion have been verified both by experiments and by astrophysical observations up to a high degree of accuracy. Because of this fact Newtonian approximation has been discussed in all text books. In "standard" general relativity (GR) described by Naralikar4, perfect fluid seems to be described as a system of particles in which the only interaction between the particles is due to gravity. The axiom of action4 is given by expression $\sum_a m_a dA$ where ma is the mass of ath particle and dA is the element of proper time along the world line of particle 'a'. In this expression pressure is not explicitly expressed. As a matter of fact pressure is due to collisions and the collision cross section depends upon electrostatic interaction between molecules or atoms and the electrostatic interaction is much stronger than gravitation. Pressure was injected into Hilbert's theory later on. Einstein starts with a perfect fluid which moves from high pressure to low pressure. The kinetic energy of the particles of the fluid gives it a relativistic mass which is different from rest mass. Einstein then argues that T44 which is the 4, 4 component of electromagnetic field is the energy of electromagnetic field and being equivalent to mass must create gravitational field. Hence he gave his initial equation as $R^{\mu\nu} = \gamma T^{\mu\nu}$ for gravity produced by electromagnetic field. Now in the case of fluid $T_4^4 = \tilde{n}$ and it is the mechanical energy of the fluid. Hence for fluid his initial equation was given as $R_\nu^\mu = \gamma T_\nu^\mu$ Later on he modified his equation to $E_\nu^\mu = \gamma T_\nu^\mu$ where $E^i = R^i - \frac{1}{2} g^i R$ so that $T_{,i}^i = 0$. But in this improved form the hydrodynamic equation $T_{,i}^i = 0$ is converted into an identity $T_{,i}^i \equiv 0$ It is interesting to note that T_4^4 is the mechanical energy of the fluid and not the total energy whereas in the electromagnetic field T_4^4 is the energy of the electromagnetic field and obviously its total energy.

On the contrarary Hilbert starts from a cluster of particles in which the only interaction is that of gravitation and hence pressure is absent.

A cluster of stars like our milky way is an example of dust because stars in the milky way do not collide and do not create pressure.The book by Narlikar4 gives $g_{ik} = \eta_{ik} + h_{ik}$ Where η_{ik} is the Minkowski space-time metric and h_{ik} denote small departure from η_{ik}

Thus $$|h_{ik}| \langle\langle 1 \quad \text{--------- (A)}$$

Naralikar says that condition A can be used to linearize the field equation of Einstein. Narlikar has not given any reference. But later on we were given a reference of book by Schutz5 to justify the field equation of Einstein. We still do not agree that condition A linearises field equation of Einstein. Because Schutz has shown that it is possible to convert a general line element into the form $ds^2 = -(1 + 2\,\Omega)(dx^2 + dy^2 + dz^2) + (1 - 2\,\Omega)\,dt^2$
----------- (5)

by transformation of coordinates and then shows that equation (5) produces Newtonian approximation. Now Eddington6 starts from equation (5) and shows that Newtonian approximation is possible only if line element can be put in the form (5) when Ω is approximate Newtonian potential. In fact Eddington has given three conditions for Newtonian approximation. One of them is the condition given by equation (5) the second is that Ω is independent of time and the third is − X,-Y,-Z = {14,4},{24,4}{34,4} i.e. the three index symbols can be interpreted as components of the field of force. Moreover Wyman has actually linearised Einstein's equation by transformation given by equation 1.4 in his paper7 and has obtained an exact solution in the transformed coordinates and finally obtained an exact solution by back transformation. Schutz consideration to construct coordinates to make calculations simplest do not apply in the case of Wyman's exact analysis.We do not agree that condition A can linearize the field equation. Field equations contain differential coefficients and we know that small term can have a big differential coefficient. In the problem of rotation one has to add to the metric an off diagonal term 2wr2Sin2q×dfdt (as shown by Wald8)

This becomes − 2wydxdt + 2wxdydt in Cartesian coordinates. The nonlinear terms arise from products of Christoffel symbols and are ≈ w2 whereas the linear terms ≈ ρ . Their ratio can be 10% in the case of rotation of Jupiter and Saturn; so that there is no linearization in this case. Further, in the case of a time dependant problem like radial motion of a star there are non-linear as well as extra linear terms. It can be shown that there are a number of non-Newtonian terms in the Hydrodynamic equation of radial motion of a star. Therefore there is no Newtonian approximation in the problem of radial motion of a star discussed by Ross9 by Einstein's theory and Kelkar et al10 by Newton's theory. Wyman's solutions are obtained by exact analysis and can exist for both strong and weak fields. From Wyman's solutions, one can get solutions for static sphere of dust and this is absurd according to Newton. As far as mathematics is concerned we agree with Narlikar that "a more satisfactory approach to Einstein's field equations is via the action principle, originally used by Hilbert (for dust)". But due to absence of Newtonian approximation we conclude that the action of Hilbert11 $\sum_a \int m_a d$ where ma is the inertial mass of the particle 'a' and da the element of proper time along the world line of 'a' is not capable of Newtonian approximation, and hence ma is probably a function of coordinates. Narlikar has not given any expression for potential energy. However, we can find expressions12 for potential energy in the G.R. of Einstein described by Eddington13.We find expression for total energy density and mechanical energy density in the G.R of Einstein, described by Eddington and Tolman14. From these two considerations potential energy density can be found. Eddington (as well as Einstein and several old authors) has defined perfect fluid as the fluid for which the components of the stress tensor are given by $P_x = P_y = P_z = P$ and the remaining components are equal to zero. Eddington states that the action vanishes (is equal to zero) in the exterior region whereas it does not exist (action cannot be found) in the interior of the source. Eddington has "proved" the Newtonian approximation of G.R. under the following conditions.

(1) The line element must be given by

$$ds^2 = -(1 + 2\,\Omega)(dx^2 + dy^2 + dz^2) + (1 - 2\,\Omega)\,dt^2$$

--------- (5)Where Ω is independent of time and obviously ρ is independent of time. Ω in (5) is the approximate Newtonian Potential because the non-linear terms given by products of

Christoffel symbols in $R_{44} \approx \left(\frac{M}{r^2}\right)^2$ and linear terms $\approx\left(\frac{M}{r^3}\right)$ The Ratio $\approx\left(\frac{M}{r}\right) \equiv 10^{-6}$ For Sun and For Earth the ratio $\approx\left(\frac{M}{r}\right) \equiv 10^{-9}$ (2) Hydrodynamic equations become Newtonian if the Christoffel's symbols {14,4}etc. can be interpreted as components of Newtonian field. It is easy to see that in the case of radial motion the g_{ik} will depend on time and in the case of rotation the Christoffel symbols cannot be interpreted as components of Newtonian field because of the term $2\,\omega r^2 \sin^2\theta\,.\,d\varphi\varphi d$ in the metric as shown earlier. Derivation of field equation by action principle shows that there are no "Constraints" under which the field equations are valid. Eddington does not mention what these constraints are but the conditions which he has given for Newtonian approximation serve as the constraints. What Hilbert has "Proved" are not Einstein's field equations but he has proved that there are no constraints under which the equations hold good.

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